Chapter 9

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Linear Momentum and Collisions

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Lecture 02

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Impulse

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Impulse and Momentum

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The momentum of a system changes if a net force from the environment acts on the system.

For momentum considerations, a system is non-isolated if a net force acts on the system for a time interval.

3

Impulse and Momentum

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- From Newton's Second Law, $\vec{F} = \frac{d\vec{p}}{dt}$
- Solving for $d\vec{p}$ gives $d\vec{p} = \vec{F}dt$
- Integrating to find the change in momentum over some time interval.

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}dt$$

The integral is called the *impulse,* $\vec{I}$, of the force acting on an object over Δt.

- A vector quantity.
- SI units are:
- $N.s = kg.m/s$

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Impulse-Momentum Theorem

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The change in the momentum of a particle is equal to the impulse of the new force acting on the particle.

$$\Delta\vec{p} = \vec{I}$$

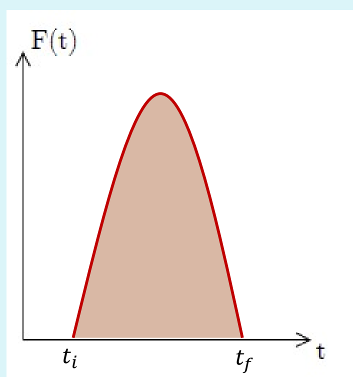
- This is equivalent to Newton's Second Law.
- This is identical in form to the conservation of energy equation.
- This is the most general statement of the principle of conservation of momentum and is called the conservation of momentum equation.
  - This form applies to non-isolated systems.
- This is the mathematical statement of the non-isolated system (momentum) model.

More About Impulse

6

The magnitude of the impulse is equal to the area under the force-time curve.

- The force may vary with time.
- Impulse is not a property of the particle, but a measure of the change in momentum of the particle.



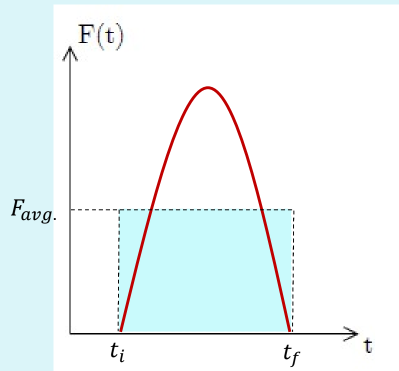
Impulse, Final

7

The impulse can also be found by using the time averaged force.

$$\vec{I} = \vec{F}_{avg.}\Delta t$$

- This would give the same impulse as the time-varying force does.



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Impulse Approximation

8

- In many cases, one force acting on a particle acts for a short time, but is much greater than any other force present.
- When using the Impulse Approximation, we will assume this is true.
  - Especially useful in analyzing collisions
- The force will be called the *impulsive force.*
- The particle is assumed to move very little during the collision.
- $\vec{p}_i$ and $\vec{p}_f$ represent the momenta *immediately* before and after the collision.

8

Momentum and Kinetic Energy

9

- Momentum and kinetic energy both involve mass and velocity.
- There are major differences between them:
  - Kinetic energy is a scalar and momentum is a vector.
  - Kinetic energy can be transformed to other types of energy.
    - There is only one type of linear momentum, so there are no similar transformations.
- Analysis models based on momentum are separate from those based on energy.
- This difference allows an independent tool to use in solving problems.

9

10

Example 23.1 The Hydrogen Atom

- The electron and proton of a hydrogen atom are separated by a distance of approximately $5.3 \times 10^{-11}\ m$. Find the magnitudes of the electric force and the gravitational force between the two particles.
- $q_e = -1.6 \times 10^{-19}\ C$

  $q_p = 1.6 \times 10^{-1}\ \ C$

  $m_e = 9.11 \times 10^{-3}\ \ kg$

  $m_p = 1.67 \times 10^{-27}\ kg$

  $k_e = 9 \times 10^{9}\ N.m^2/C^2$

  $G = 6.67 \times 10^{-11}\ N.m^2/kg^2$

10

1-D impulse

Saturday, 30 January, 2021 15:25

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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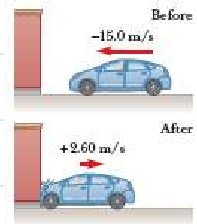
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

In a particular crash test, a car of mass $1500\ kg$ collides with a wall. The initial and final velocities of the car are $\vec{v}_i = -15\ \hat{\imath}\ m/s$ and $\vec{v}_f = 2.6\ \hat{\imath}\ m/s$, respectively. If the collision lasts $0.15\ s$, find:

- the impulse caused by the collision and
- the average net force exerted on the car.



$$I = m\left(v_f - v_i\right)$$
$$= 1500 \times (26 - {}^{-}15) = 61500\ N \cdot s$$

$$I = F_{avg}\,\Delta t$$
$$F_{avg} = \frac{I}{\Delta t} = \frac{61500}{0.15} = 4.1 \times 10^5 N$$

Impulse - Area

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The magnitude of the net force exerted in the x direction on a $2.5\ kg$ particle varies in time as shown. Find:

- the impulse of the force over the $5\ s$ time interval,
- the final velocity the particle attains if it is originally at rest,
- its final velocity if its original velocity is $-2\ \hat{\imath}\ m/s$, and
- the average force exerted on the particle for the time interval between 0 and $5\ s$.

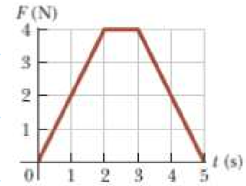

Figure P9.19

P9.19 (a) The impulse is in the *x* direction and equal to the area under the *F-t* graph:

$$I=\left(\frac{0+4\text{ N}}{2}\right)(2\text{ s}-0)+(4\text{ N})(3\text{ s}-2\text{ s})+\left(\frac{4\text{ N}+0}{2}\right)(5\text{ s}-3\text{ s})$$
$$=12.0\text{ N}\cdot\text{s}$$
$$\boxed{\vec{\mathbf{I}}=12.0\text{ N}\cdot\text{s }\hat{\mathbf{i}}}$$

(b) From the momentum-impulse theorem,

$$m\vec{\mathbf{v}}_i+\vec{\mathbf{F}}\Delta t=m\vec{\mathbf{v}}_f$$
$$\vec{\mathbf{v}}_f=\vec{\mathbf{v}}_i+\frac{\vec{\mathbf{F}}\Delta t}{m}=0+\frac{12.0\,\hat{\mathbf{i}}\text{ N}\cdot\text{s}}{2.50\text{ kg}}=\boxed{4.80\,\hat{\mathbf{i}}\text{ m/s}}$$

(c) From the same equation,

$$\vec{\mathbf{v}}_f=\vec{\mathbf{v}}_i+\frac{\vec{\mathbf{F}}\Delta t}{m}=-2.00\,\hat{\mathbf{i}}\text{ m/s}+\frac{12.0\,\hat{\mathbf{i}}\text{ N}\cdot\text{s}}{2.50\text{ kg}}=\boxed{2.80\,\hat{\mathbf{i}}\text{ m/s}}$$

(d) $\vec{\mathbf{F}}_{\text{avg}}\Delta t=12.0\hat{\mathbf{i}}\ \text{N}\cdot\text{s}=\vec{\mathbf{F}}_{\text{avg}}(5.00\text{ s}) \rightarrow \vec{\mathbf{F}}_{\text{avg}}=\boxed{2.40\hat{\mathbf{i}}\text{ N}}$

Impulse - Integration

Saturday, 30 January, 2021 15:27

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Starting from rest, a $65\ kg$ athlete jumps down onto a platform from a height of $0.6\ m$. While she is in contact with the platform during the time interval $0 < t < 0.8\ s$, the force she exerts on it is described by the function: $F(t) = 9200t - 11500t^2$, where F is in Newtons and t is in seconds.

- What impulse did the athlete receive from the platform?
- With what speed did she reach the platform?
- With what speed did she leave it?
- To what height did she jump upon leaving the platform?

P9.20 (a) A graph of the expression for force shows a parabola opening down, with the value zero at the beginning and end of the 0.800-s interval. We integrate the given force to find the impulse:

$$\begin{aligned}
I &= \int_0^{0.800\text{s}} F\,dt \\
&= \int_0^{0.800\text{s}} (9\,200\ t\ \text{N/s} - 11\,500\ t^2\ \text{N/s}^2)\,dt \\
&= \left[\frac{1}{2}(9\,200\ \text{N/s})t^2 - \frac{1}{3}(11\,500\ \text{N/s}^2)t^3\right]_0^{0.800\text{s}} \\
&= \frac{1}{2}(9\,200\ \text{N/s})(0.800\ \text{s})^2 - \frac{1}{3}(11\,500\ \text{N/s}^2)(0.800\ \text{s})^3 \\
&= 2\,944\ \text{N}\cdot\text{s} - 1\,963\ \text{N}\cdot\text{s} = 981\ \text{N}\cdot\text{s}
\end{aligned}$$

The athlete imparts a downward impulse to the platform, so the platform imparts to her an impulse of $\boxed{981\ \text{N}\cdot\text{s, up.}}$

(b) We could find her impact speed as a free-fall calculation, but we choose to write it as a conservation-of-energy calculation:

$$mgy_{\text{top}} = \frac{1}{2}mv_{\text{impact}}^2$$

$$v_{\text{impact}} = \sqrt{2gy_{\text{top}}} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(0.600\ \text{m})}$$
$$= \boxed{3.43\ \text{m/s, down}}$$

(c) Gravity, as well as the platform, imparts impulse to her during the interaction with the platform

$$I = \Delta p$$
$$I_{\text{grav}} + I_{\text{platform}} = mv_f - mv_i$$
$$-mg\Delta t + I_{\text{platform}} = mv_f - mv_i$$

solving for the final velocity gives

$$v_f = v_i - mg\Delta t + \frac{I_{\text{platform}}}{m}$$
$$= (-3.43\ \text{m/s}) - \left(9.80\ \text{m/s}^2\right)(0.800\ \text{s}) + \frac{981\ \text{N}\cdot\text{s}}{65.0\ \text{kg}}$$
$$= \boxed{3.83\ \text{m/s, up}}$$

Note that the athlete is putting a lot of effort into jumping and does not exert any force "on herself." The usefulness of the force platform is to measure her effort by showing the force she exerts on the floor.

(d) Again energy is conserved in upward flight:

$$mgy_{\text{top}} = \frac{1}{2}mv_{\text{takeoff}}^2$$

which gives

$$y_{\text{top}} = \frac{v_{\text{takeoff}}^2}{2g} = \frac{(3.83\ \text{m/s})^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{0.748\ \text{m}}$$